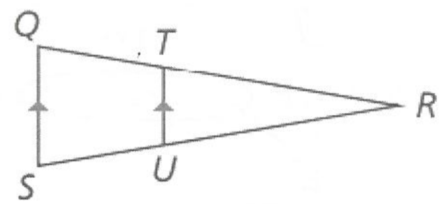
Name: Date: Period: 6

Proportionality Theorems

son Objective IWBAT use the Δ proportionality Thm and its converse; use other proportionality Thms

Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

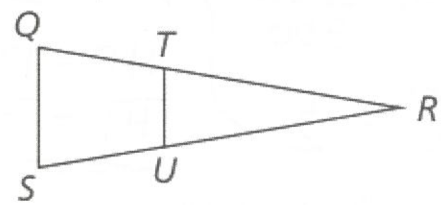


if $\overline{QS} \parallel \overline{TU} \rightarrow \frac{RT}{QT} = \frac{RU}{SU}$

a b

Converse of the Triangle Proportionality Theorem

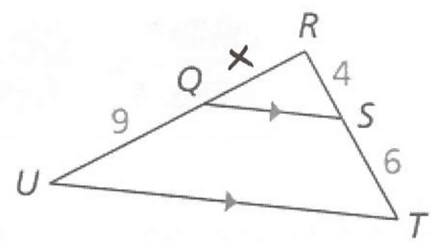
If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



If $\frac{RT}{QT} = \frac{RU}{SU} \rightarrow \overline{QS} \parallel \overline{TU}$

mple:

1. In the diagram, $\overline{QS} \parallel \overline{UT}$, RS = 4, ST = 6, and QT = 9. What is RQ?



$\frac{x}{9} = \frac{4}{6} \rightarrow 6x = 36$

$\boxed{x = 6}$ $\boxed{RQ = 6}$

BRIEF REVIEW: Which statements, formed by an original conditional statement, are logically equivalent?

➔ The conditional ($a \rightarrow b$) and the contrapositive ($\sim b \rightarrow \sim a$)

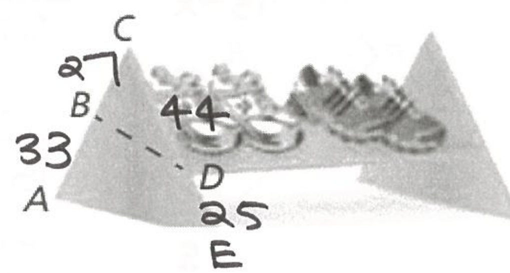
➔ The converse ($b \rightarrow a$) and the inverse ($\sim a \rightarrow \sim b$)

The theorems in the solid boxes above imply the following:

Contrapositive of the Triangle Proportionality Theorem	**Inverse of the Triangle Proportionality Theorem**
If $\frac{RT}{QT} \neq \frac{RU}{SU} \rightarrow \overline{QS} \nparallel \overline{TU}$	If $\overline{QS} \nparallel \overline{TU} \rightarrow \frac{RT}{QT} \neq \frac{RU}{SU}$

Example:

2. On the shoe rack shown, BA = 33 cm, CB = 27 cm, CD = 44 cm, and DE = 25 cm. Explain why the shelf is not parallel to the floor.

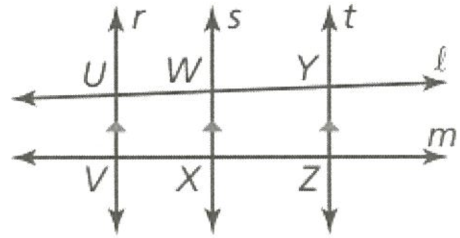


$\frac{27}{33} = \frac{44}{25}$ $(27)(25) = (33)(44)$

$675 \neq 1452$

Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.



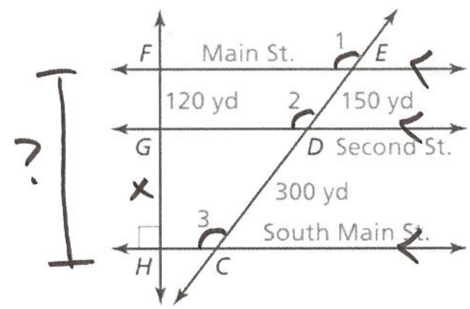
$$\frac{UW}{WY} = \frac{VX}{XZ} \quad \begin{array}{l} \leftarrow \text{btwn } r \,\&\, s \\ \leftarrow \text{btwn } s \,\&\, t \end{array}$$

$\ell \qquad m$

Example:

3. In the diagram, $\angle 1 \cong \angle 2 \cong \angle 3$. GF = 120 yd, DE = 150 yd, and CD = 300 yd. Find the distance HF between Main Street and South Main Street.

$\overline{EF} \parallel \overline{DG} \parallel \overline{CH}$ since corresp. $\angle$s $\cong$



$$\frac{120}{x} = \frac{150}{300}$$

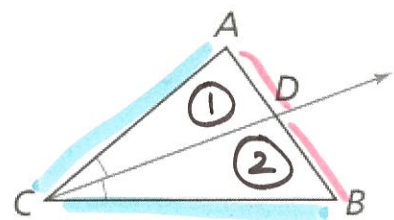
$$\frac{120}{x} = \frac{1}{2} \rightarrow x = 240$$

$HF = 120 + 240$

$\boxed{HF = 360 \text{ yd}}$

Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

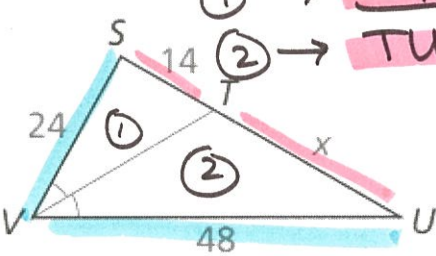


$$\frac{AD}{DB} = \frac{CA}{CB} \quad \begin{array}{l} \leftarrow \text{same } \triangle\ (\triangle 1) \\ \leftarrow \text{same } \triangle\ (\triangle 2) \end{array}$$

Example:

4. Find the value of x.

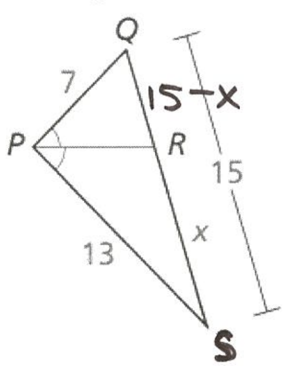
$$\begin{array}{l} ① \rightarrow \\ ② \rightarrow \end{array} \frac{ST}{TU} = \frac{VS}{VU}$$



$$\frac{14}{x} = \frac{24}{48}$$

$$\frac{14}{x} = \frac{1}{2} \rightarrow \boxed{x = 28}$$

5. In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find RS.



$$\frac{RS}{QR} = \frac{PS}{PQ}$$

$$\frac{x}{15-x} = \frac{13}{7}$$

$7x = 13(15-x) \rightarrow 7x = 195 - 13x$

$20x = 195$

$\boxed{x = 9.75}$